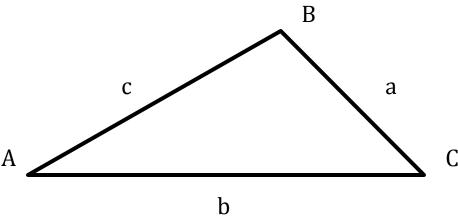
Given  $\triangle ABC$  is **not** a right triangle. The length of a given side  $\boldsymbol{a}$  can be determined by the Law of Cosines:  $a^2 = c^2 + b^2 - 2bcCos A$ 



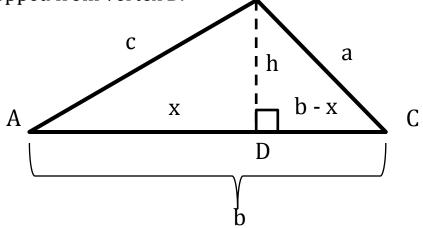
## **Proof**

A perpendicular auxiliary can be dropped from vertex B.

Also note that,

$$Sin A = \frac{h}{c} \Rightarrow h = cSin A$$

$$Cos A = \frac{x}{c} \Rightarrow x = cCos A$$



В

Deriving the Law of Cosines	
STEPS	REASONS
$a^2 = h^2 + (b - x)^2$	Applying the Pythagorean Theorem to $\triangle$ <i>CBD</i>
$a^2 = (cSin A)^2 + (b - cCos A)^2$	Substituting for $h$ and $x$
$a^2 = (c^2 Sin^2 A) + (b^2 - 2bcCos A + c^2 Cos^2 A)$	Simplifying by multiplying the squared-binomial
$a^2 = (c^2 Sin^2 A + b^2 - 2bcCos A + c^2 Cos^2 A)$	Simplifying
$a^2 = c^2(Sin^2A + Cos^2A) + b^2 - 2bcCosA$	Simplifying by factoring out the common factor $c^2$ and rewriting using the Associative Property of Addition
$a^2 = c^2(1) + b^2 - 2bcCos A$	Simplifying using the Pythagorean Identity $Sin^2A + Cos^2A = 1$
$a^2 = c^2 + b^2 - 2bcCos A$	Simplifying

The same process can be used to find the length of b and c.